# THE USE OF CONCEPT MAPPING PROCEDURE TO CHARACTERISE TEACHERS' MATHEMATICAL CONTENT KNOWLEDGE

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*This paper describes the use of a method for analysing concept map data to generate information on quantitative and qualitative features of a mathematics teacher's knowledge base in the area of geometry. The system of analysis used in this study provides a useful way to represent the state and connectedness of teachers' knowledge in the area of geometry and trigonometry. Use of this system indicates that this experienced teacher has a rich store of knowledge which is complete and accurate. His knowledge base shows a high degree of branching with depth of knowledge within each branch.*

## INTRODUCTION

Many recent discussions of the role of the mathematics teacher emphasise the importance of teachers helping students to develop effective knowledge structures, representations of mathematical content that will allow the student to productively explore a suitable range of mathematical problems. The sense of this perspective is clearly articulated in the recommendations such as that of the National Council of Teachers of Mathematics (1989, p. 128) that there is a need for teachers to 'shift from dispensing information to facilitating learning'. Knapp (1997) views the adoption by teachers of this shift in conceptualisation of teaching as one of the central planks of the broad movement to generate systemic reform in mathematics and science teaching. This change in emphasis for teachers' actions recognises the crucial role that teaching plays in influencing what knowledge is constructed by students. Even though the ultimate control of the knowledge construction process rests with the student, it is clear that the actions of the teacher in a lesson have substantial impact on the outcome of that process.

In the wider project in which this study is embedded we are interested in drawing links between the knowledge and actions of mathematics teachers and the mathematical knowledge and actions of their students. In undertaking this research one of our major assumptions is that the use of connecting, or linking, actions by the teacher is affected to a substantial extent by the state of connectedness of the teacher's own knowledge. Thus one of our first concerns has been to attempt to develop a procedure for representing that state of connectedness. The purpose of this paper is to describe the procedures we have developed to represent the quality of connectedness of teachers' mathematical knowledge. We will illustrate the procedures by focussing on one teacher's knowledge of a) the schema of square and b) relations between a square and a number of other geometrical schemas.

### Knowledge Connectedness

An important consideration in the study of mathematical knowledge is the degree of connectedness that exists among the various pieces of that knowledge. Labels for the structured representations vary between research programs but it is common for the "packets of integrated information" (Hunt, 1993, p. 530) to be referred to as schemas, or schemata. When viewed at a larger grain size, a network of schemas can be thought of as *mental spaces* , structured "conceptual packets constructed as we speak, for purposes of local understanding and action"(Fauconnier & Turner, 1998, p. 137) The quality of connections among knowledge components is assumed to influence the ease with which the presence of one element aids in the retrieval and use of another in a problem environment.

Mayer (1975) made use of the notion of connectedness in his description of the accumulation of new information in long term memory as adding new 'nodes' to memory and connecting the new nodes with components of the existing network. 'Internal connectedness' was referred to the degree to which new nodes of information were connected with one another to form a single well-defined structure. The degree to which new nodes of information were connected with information already existing in the learner's cognitive structure, Mayer referred to as 'external connectedness.'

In a further analysis of mathematical knowledge structure, Greeno (1978) noted that within a schema the learner could also be expected to establish connections of how the schema relates to specific problems and situations. This issue was examined further by Chinnappan (1998a, 1998b) in which he found evidence to support the view that the linking of the different pieces of knowledge of geometry and trigonometry into schemas could play an influential role in the construction of appropriate representations for problems. Use of the term connection by both Mayer and Greeno also involved an attempt to establish a link between knowledge of a schema and knowledge of how to use that schema, thus being concerned with the proceduralization of declarative knowledge.

We have used the notions of internal (within-schema) and external (between-schema) connectedness as the core of a conceptual framework for representing the qualitative characteristics of the teachers' knowledge. We see this as useful for two main reasons. First, it allows us to represent the complexity of a knowledge base in a way that focuses on the state of organisation of that knowledge. The connectedness framework also provides a basis for establishing relationships between knowledge states and teaching actions. In relation to this second purpose we suggest that one of the key motivations of the teachers, and one of the most important sets of their actions, can be seen as attempting to facilitate the establishment of both internal and external connections in the knowledge bases of the students. In this way we argue that it makes sense to seek out such relationships in the teachers' knowledge base, for it seems reasonable to expect that it is from these connections that teachers will generate actions that are designed to improve the state of connectedness of the students' knowledge base.

## Representation of Structural Knowledge by Concept Maps

As the study of connectedness involves the examination and specification of schemas and relations that teachers have constructed over a period of time, we need tools that would help us represent the organisational features of teachers' knowledge base. A number of different mapping procedures have been used for this purpose including concept maps. Concept maps, in general, are graphs consisting of nodes and labelled lines. The nodes are used to indicate concepts while the lines correspond to a relation between pairs of concepts. The label on the line tells how the two concepts are related by the individual. Shavelson, Lang and Lewin (1993) referred to this relation as a proposition and argued that concept maps represent some important aspect of learner's propositional knowledge in a domain. In this sense, a concept map could be used to probe teachers' understanding of not only relationships among concepts but also information about how they perceive and use those concepts to explain other related concepts.

Despite their popularity in teaching and research, attempts to use concept maps to represent the quality of knowledge organisation in quantitative terms have, so far, not been very successful (Lawson, 1994). It seems, therefore, that the potential of a concept mapping procedure for representing knowledge organisation can be further exploited and in this paper we have attempted to realise more of this potential by developing a range of indices of knowledge organisation from a teacher's concept maps. In using this form of map we are not intending to suggest that we have captured an enduring representation of an individual's conceptual network. We assume that all conceptual networks are dynamic mental spaces, so that we are representing the conceptual space that has been constructed across the times of our interaction with the participant.

Williams (1998) examined the value of concept maps as instruments for the assessment of conceptual knowledge of a group of professional mathematicians and students. In that study participants were asked to generate a list of terms about functions and organise them into maps. Each of the maps contained function-related concepts in ovals, with words indicating relationships among concepts appearing on the line connecting concepts. The maps showed evidence of participants' relating concepts in the form of chains. The maps, however, were not designed to allow for the representation of cross linking between items of information between chains. Further, Williams did not attempt to represent the quality of conceptual understanding. Her analysis was also designed to be more global, as it allowed her to make judgments about the algorithmic nature of students' understanding as opposed to deeper level or categorical understanding shown by the experts. Therefore, although William's(1998) study showed the value of use of concept maps in the study of conceptual knowledge it suffered from the lack of detail about the various levels of connections and their influence on conceptual understanding.

In the present study we were interested in describing the quantity and quality of the organisation of the teacher's knowledge base about the topic of geometry in more detail. In doing so we were interested in representing the configuration of the connections that teachers have built among these concepts. Unlike that of Williams's(1998) study, our design used more than one context within which participants could externalise their knowledge. It was, therefore, reasonable to expect the concept maps to be more varied and complex. Thus, the structure of the concept map for the present study was designed to allow for the visualisation of the hierarchical organisation of geometric concepts and subconcepts as well as lateral links among these concepts and information about the teaching of the concepts. We will now describe the design and development of a concept mapping procedure that can be used to characterise the organisational state of teachers' knowledge of key concepts in geometry and trigonometry.

## PROCEDURE

The teacher whose knowledge is the subject of this report was one of five experienced teachers recruited, along with five novice teachers, for the larger project. The experienced teachers were selected on the basis of two criteria: First they had at least 15 years of mathematics teaching experience at the high school level; and second, they were all recommended by their peers or professional subject associations as exemplary teachers. The concept map developed for this report was based on the knowledge exhibited by one expert teacher, Gary. In addition to having 20 years of teaching experience, Gary was the head of the mathematics department at his high school and was also involved in the writing of mathematics textbooks for high school students.

Gary was interviewed individually during school hours and his responses were audio- and video-recorded for later transcription. Gary was told that the purpose of the study was to find out what teachers might know about topics in geometry and the teaching of these topics.

Three interviews were conducted, each lasting about one hour. During the first interview Gary was asked to talk about a list of focus schemas in the areas of geometry, trigonometry, and coordinate geometry. The list included 13 focus schemas which were required to solve the four problems given to the teachers during the second interview, and six related schemas.

The 13 focus schemas were squares, rectangles, lines, similar triangles, congruent triangles, parallel lines, area, coordinates, triangles, right-angled triangles, regular hexagons, regular octagons, and circles. The six related schemas were plane figures, trapezium, polygons, rhombus, quadrilateral, parallelogram. Gary was invited to use diagrams to explain his thoughts if he wished to do so.

During the second interview, he was asked to solve four problems, each of which required use of schemas in the above three areas. Gary was asked to talk aloud as much as possible during the problem-solving attempts . During the final interview Gary was required to expand on items that he mentioned during the first interview when he freely recalled information.

In addition to the experienced teachers recruited for the project information was also sought from an academic mathematician. This mathematician was a member of a university mathematics department whose area of professional expertise was in the area of geometry. The focus of our interview with this mathematician was generation of a list of features and relationships for each of the focus schemas. For this study the list of features and schemas related to the focus schema square developed from this interview was used in evaluating the completeness and accuracy of the teacher's knowledge base.

## Structure of the Concept Map

A primary consideration in the design of the concept maps was to capture the detail of schemas that teachers have acquired and relations that they have built within and between these schemas. The boxes and circles, or nodes, in the concept maps indicate schemas and features and the lines joining the boxes/circles show that a relationship was expressed between schema and features or between schemas. We identified four types of information about a focus schema and the relations that had been built around that schema.

1. *The defining features of the focus schema*

2. *The related features of the focus schema*

3. *Relationships between the focus schema and other schemata*

4. *Other representations of the focus schema*

## Scoring of Concept Maps

In scoring the concept maps we adopted a strategy that would enable us generate a comprehensive description of the qualities of connectedness of teachers' knowledge. The connectedness of knowledge can be described both in terms of the number of knowledge components present as well as in terms of the qualitative relations that exist among the knowledge components. In our concept maps the qualitative character of the knowledge base refers to the pattern and extent of connections that were constructed by the teachers.

The basic units of analysis used to construct the maps were nodes and links. Nodes were established for all schemas associated with geometric terms or features, or attributes, associated with these terms that were mentioned by the participant in any of the tasks. The task on which the node was first identified was also recorded on the map. Links were interpreted as propositions. All propositions expressed in words or actions by the participant in any of the tasks were recorded, along with any labels expressed for the linking relationship. Nodes and links were established by consensus using two coders. The measures used to score maps and their definitions are available from the authors.

The following measures were derived for each concept map:

*1. Quantity*

We interpreted quantity as having two sub-categories:(a) number of nodes , and (b) number of links.

*2. Quality*

The quality of the concept maps were analysed for the two broad dimensions of integrity and connectedness. Integrity was analysed in terms of (a) completeness and (b) accuracy. The idea of integrity of knowledge was not used here in an absolutist sense. In scoring teachers' responses for completeness and accuracy we used the list of ideas generated the expert mathematician as a reference point. For completeness our scoring was not bounded by this list. If teachers produced a node or link that was not found in the mathematician's list but one that was mathematically acceptable this was also identified and scored.

Accuracy refers to the degree of correctness of the information provided by teachers. Information that is not correct may be manifest itself in various forms. An incorrect piece of knowledge could be a misconception (McKeown and Beck, 1990) or it could be 'underdifferentiated' or 'garbled' (Perkins and Simmons, 1988). For example, some students may not be able to differentiate the attribute of the schema of area and the measurement of area. This would be a case of a misconception. An example of garbled knowledge could be: 'Diagonals of a square are lines of symmetry for squares. Therefore, diagonals of rectangles are lines of symmetry'.

Connectedness was represented by four sub-categories: (c) depth, (d) branching, (e) crosslinking and (f) quality of relationships. Depth here refers to the extension of connections in a concept map in vertical directions along a single path. Within schema depth is a measure of the degree of vertical connection in a schema. We refer to this spread as occurring over a different vertical layers of the nodes in the concept map. Between schema depth is a measure of vertical connections among schema.

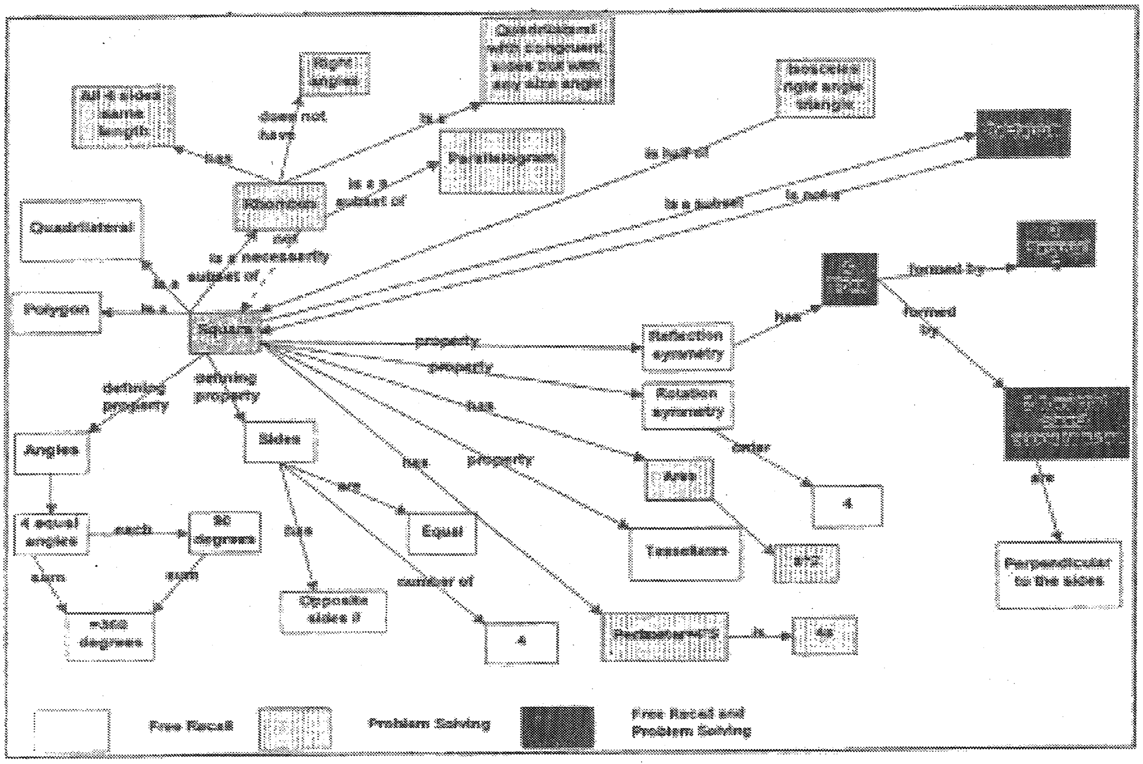
Branching is a measure of the number of paths associated with a feature. Crosslinking is a measure of horizontal linking between paths or between schemas. The quality of relationship measure attempts to rate the complexity of the label given to a particular link. For the purposes of scoring the map, we found it convenient to divide the organisational features of the information into two types of schematic links: within schema and between schema (Marshall, 1995).

## RESULTS AND DISCUSSION

### Gary's Concept Map

Figure 1 shows the concept map that was constructed for the focus schema of square for our expert teacher, Gary. The most striking impression of this map was that Gary has built up a large number of connections involving squares, as shown by the number of arrows emanating to and from the Focus schema. The bottom left hand corner of Figure 1 showed that his knowledge about the properties of squares is extensive, for both defining and related features. The top-half of the map indicated that Gary has been able to link squares with other polygons such as rhombus and isosceles triangles. Gary's understanding of defining properties of a square was extended to more advanced features such as symmetry and area. This component of his knowledge is depicted by the right-half of Figure 1. The upper left section of the concept map provides information relevant to the contexts in which the above knowledge components were activated and the ways that these would be expressed by this teacher.

*Figure 1*
*Gary's Concept Map for a Square*




The concept map of this experienced teacher revealed that, in a quantitative sense, he has a rich store of knowledge as manifested by the multiple links he had constructed with many facets of a square. These links suggests that the teacher not only has a well-developed understanding about the properties of square but more importantly, he has an extensive network of connections between squares and other geometrical figures. For example, he was able to represent a square as a polygon and as a quadrilateral. He saw square as part of a rhombus but noted correctly that a rhombus was not a square. His ability to recognise a right-angled triangle as forming part of the square also indicated that the teacher was able to use basic properties such as right-angles to create other figures. Gary's knowledge base indicates that he was able to highlight differences and similarities that exist between square and related figures.

Gary commented that a square has symmetrical properties and that this property can be created by more than one way. This understanding of symmetry is extended to his observations that squares tessellate. Here one can detect not only an understanding of the geometric properties but also practical applications of squares.

## Within-schema Ratings

Results of analysis for the Focus schema of square in terms of quantity and quality of the links are provided in Tables 1 and 2. Table 1 shows connections made by Gary, that are internal to square, links that indicate within schema organisation. Table 1 shows that Gary's concept map contained a total of nine defining, and 12 nodes associated with the related features of the focus schema. Table 1 also shows that the number of links was almost the same as the number of defining and related features nodes.

*Table 1*
*Analysis of Concept Map for a Square: Within Schema*

| **FEATURES** | **QUANTITY** | | **QUALITY** | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Integrity | | Connectedness | | | | | |
| | Number of nodes | Number of links | Completeness | Accuracy | Depth | Branching | Cross-linking | Quality of relationships | | |
| | | | | | | | | Simple | Moderate | Complex |
| **Defining Features** | | | | | | | | | | |
| Angles | 5 | 5 | | | Level 3 | Moderate | None | 2 | 1 | |
| Sides | 4 | 4 | | | Level 2 | Moderate | None | 3 | | |
| Subtotal | 9 | 9 | High | High | | | | 5 | 1 | 0 |
| **Related Features** | | | | | | | | | | |
| Area/ Perimeter | 4 | 4 | | | Level 2 | Moderate | None | | 2 | |
| Symmetry | 7 | 7 | | | Level 3 | High | None | | 2 | |
| Tesselates | 1 | 1 | | | Level 1 | None | None | 1 | | |
| Subtotal | 12 | 12 | High | High | | | | 1 | 4 | 0 |
| **Applications** | | | | | | | | | | |
| | | | | | | | | | | |
| Subtotal | 0 | 0 | Low | x | | x | x | 0 | 0 | 0 |
| **TOTAL** | **21** | **21** | | | | | | **6** | **5** | **0** |

In the qualitative analysis there is, first, a high level of integrity in the defining and related features components of Gary's map when judged against the knowledge base of the expert mathematician, as indicated by high scores for both accuracy and completeness. However, there were variations in the levels of depth of links of his knowledge base across the defining and related features of the focus schema . There was a greater depth of connectedness for angles and symmetry than for tessellation, area or perimeter. Secondly, there is a low level of completeness in the applications of the square. Because Gary did not present an example of how the square can be applied in the real world, we were unable to make any judgements about the accuracy of his knowledge about the applications of squares.

Gary's map showed high levels of branching especially for several features, with tessellation again being the least developed feature. Figure 1 shows that he can differentiate between rotational symmetry and reflective symmetry, sides and angles, and area and perimeter. Within each of the two symmetry branches, the knowledge is well structured and organised into a highly integrated chunk. However, the lack of crosslinking between these two symmetry branches indicates that there seems to be little integration between knowledge about line symmetry and rotational symmetry.

The pattern of scores in the last three columns of Table 1 indicate that the quality of the descriptions used by this teacher to label the relationships among nodes defining and related to the focus schema were either simple or moderate. For example, Gary mentioned that an isosceles triangle is half of the focus schema, square. This is a correct statement and there was an attempt to elaborate this relationship, as indicated by the comments he made about the relative size of the two figures. However, he did not go further and establish a bidirectional connection about how a square might be related to the isosceles triangle, or discuss the implications of such relationships in using or deriving, say, the Pythagoras' theorem.

Overall, Gary's within schema knowledge about square appears to be extensive and is well differentiated across several layers and along several branches. However, the level of integration between the different branches of knowledge about the square could only be described as minimal, as evidenced by the scores for crosslinking.

## Between Schema Ratings

Table 2 shows the ratings given to the various dimensions of Gary's knowledge about square and other focus schemas. The quantitative analysis showed that Gary had made links between the focus schema (square) and four other schemas. Of these the highest number of links and nodes involved the rhombus schema, followed by rectangle. The qualitative analysis indicates that the links made were complete and accurate, suggesting a high level of integrity about this aspect of his knowledge base. With regard to depth, there were fewer instances of spread of links at level 2 or higher than in his within-schema knowledge structure. The links at the highest depth was between square and rhombus, rhombus and parallelogram, and rhombus and quadrilateral, thus indicating the existence of a hierarchical understanding of these schemas.

Gary provided two instances of simple relationships and two complex relationships in his discussion of these schemas. For instance, he mentioned that a square is a subset of a rhombus but not all rhombuses are squares suggesting an understanding of the complexity of relations between the two geometrical figures. He further commented that all squares are rectangles but not vice versa, again indicating a complex relationship about differences and similarities between the two figures.

*Table 2*
*Analysis of Concept Map of a Square: Between Schema*

| FEATURES | QUANTITY | | QUALITY | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Integrity | | Connectedness | | | | | |
| | Number of nodes | Number of links | Completeness | Accuracy | Depth | Branching | Cross-linking | Quality of relationships | | |
| | | | | | | | | Simple | Moderate | Complex |
| Quadri-lateral | 1 | 1 | | | Level 2 | | | 1 | | |
| Rhombus | 5 | 6 | | | Level 2 | | | | | 1 |
| Rectangle | 1 | 2 | | | Level 1 | | | | | 1 |
| Isosceles right-angled triangle | 1 | 1 | | | Level 1 | | | 1 | | |
| Polygon | 1 | 1 | | | Level 1 | | | 1 | | |
| | | | | | | | | | | |
| **TOTAL** | **9** | **11** | High | High | | Yes | None | **3** | **0** | **2** |

In summary an analysis of the quantitative and qualitative dimensions of the knowledge shows that Gary has a rich set of connections with evidence of complex differentiation in some instances. Overall, his knowledge has high integrity and shows evidence of substantial branching in certain areas. As an experienced teacher it might have been expected that Gary would have shown evidence of more crosslinks between branches. Half of the relations that were discussed by Gary could be categorised as complex in nature. In general, however, Gary's knowledge network is not well integrated between branches. This lack of integration between the different branches of knowledge from an experienced teacher may influence his teaching in ways that students may not be able to draw out important differences between key features about squares and other geometry figures.

## CONCLUSION

The purpose of this paper was to describe the use of a method for analysing concept map data for generating information on quantitative and qualitative features of a mathematics teacher's knowledge base in the area of geometry. By way of illustration we described the integrity and the levels of connectedness of the knowledge one teacher has built up around the concept of square. The representation of the teacher's knowledge was built up from his involvement in three different performance contexts in each of which the teacher could have displayed his understanding of square and its applications.

We contend that the system of analysis used in this study takes us beyond that used in other studies that have employed concept mapping or similar graphical systems as a means of representing knowledge states. The current system takes us beyond the point reached by Williams (1998), enabling us to present more detailed and differentiated description of this teacher's knowledge base in this area of geometry.

Use of this system indicates that this experienced teacher has a rich store of knowledge which is complete and accurate. His knowledge base shows a high degree of branching with depth of knowledge within branching. The relationships between features and schemas shows that he does have a range of complexity. Analysis also suggest that the degree of crosslinking could be higher.

Overall, we have delineated a set of dimensions to describe a subject-matter knowledge in a quantitative and qualitative sense. We have identified areas where the quality of this teacher's knowledge could be improved. The relatively low levels of crosslinking could have implications for the quality of explanations provided, and it could also influence the accessing of knowledge.

Experience with this procedure suggests that there is value in use of the idea of connectedness to make judgements about the qualitative features of a knowledge network. The procedures we have developed here extend the utility of Mayer's (1975) distinction between internal and external connectedness. The levels of connectedness of this framework allowed us to make a distinction between the richness of the internal and external connections associated with square that were made by this teacher. In a practical sense this analysis seems to have provided a way for use to establish a qualitative estimate relevant to the teacher's knowledge base that can be used in other parts of the project where relationships between mathematical content knowledge, pedagogical knowledge and teaching actions are of concern.

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